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Impacts of Speedup of Market System
on Price Formations
using Artificial Market Simulations

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Abstract

Recently, financial exchange (market) systems are being made speedup because of competition between Markets and big investors' demands. It is said that the speedup of market systems is good for liquidity by increasing providing liquidity traders, on the other hand, it is also said that the speedup is bad for system costs to exchange (market) providers and investors.

Therefore, in this study we investigated price formations and market efficiency for various "latency"(length of time required to transport data), other settings were exactly same, by artificial market simulations. We tried to clarify mechanisms of those and discussed how much speedup is best for market efficiency. In the case of the latency is larger than the order interval, when market trend has been finished, the agents cannot quickly change their estimate prices, and the agents make unnecessary market following trades, finally, the execution rate becomes larger. And we showed that increasing execution rate reduces limit orders, and this makes a bid ask spread wider, then a market becomes less efficient. This indicates that it is needed that a market is efficient; the latency should be sufficiently smaller than the order interval.

We also analyzed empirical data of Tokyo Stock Exchange, and compared empirical results with simulation results. Tokyo Stock Exchange was possible that the market was chronically inefficient, large portion of trading time, before arrowhead by the mechanism simulation results showed. On the other hand, the market is not chronically inefficient by the mechanism at least for a time scale of minutes.

The biggest contribution of this study is an indication of possibly that a large latency would directly make a market price formation inefficient. We did not directly investigate effects of high frequency traders (HFTs), and this is a future work.

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1 Introduction

Recently, financial exchange (market) systems are being made speedup because of competition between Markets and big investors' demands. It is said that the speedup of market systems is good for liquidity by increasing providing liquidity traders[*1]. On the other hand, it is also said that the speedup is bad for system costs to exchange (market) providers and investors. It is most important factor for the speedup that "latency"; length of time required to match orders and to transport data. Latency is smaller, a market system is faster. Tokyo Stock Exchange launched "arrowhead" which is faster market system than previous one from 2010, and there are many empirical studies to compare price formations before and after arrowhead launched[*2].

It is very difficult to discuss dramatically changing financial market system such as latency by only using results of empirical studies. This is because so many factors cause price formation in actual markets that an empirical study cannot isolate the pure contribution of the changing system to price formation, and because it is impossible to conduct experiments for the changing system in real financial markets and to treat situations which have never occurred before in real financial markets.

An artificial market, which is a kind of a multi agent simulation, can isolate the pure contribution of these changing system to the price formation and can treat the changing that have never been employed [*3]. These are strong points of the artificial market simulation study.

Agents (general investors) and a price determination mechanism (a financial exchange) are explicitly modeled in artificial markets. Both of them are built in a computer, and we can simulate price formations only by the computer; actual financial data is not needed. Artificial market simulation studies contributed to replicate statistical behavior of price formations observed in actual financial market, and to discuss micro-macro interaction mechanisms of financial bubbles and crises which are very difficult for other methods to discuss.

Furthermore, recently, an estimation of market impacts[*4], effects of hedge trading between underlying securities and derivatives[*5], and regulations and systems of financial markets[*6] were investigated by artificial market simulations.

A few recent simulation studies give us valuable knowledge to change actual financial market systems or regulations, for example, Mizuta et al. (2013b) investigated tick sizes (a minimum unit

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[*1] For example, Angel et al. (2015).

[*2] For example,Uno and Shibata (2011).

[*3] Excellent reviews; LeBaron (2006); Chen et al. (2012); Cristelli (2014).

[*4] For example, Oesch (2014).

[*5] For example, Ohi et al. (2011); Kawakubo et al. (2014).

[*6] For example, Westerhoff (2008); Yagi et al. (2010); Yeh and Yang (2010); Kobayashi and Hashimoto (2011); Thurner et al. (2012); Mizuta et al. (2013a, 2014a,b).

of a price change) which were actually changed smaller on Tokyo Stock Exchange in 2014[*7].

About speedup of market systems, effects to price formations by high frequency traders (HFTs) existing[*8], and effects by arbitrage trading between markets which have different latency were investigated by artificial market[*9] simulations.

However, no artificial market simulation study investigated a pure effect only by difference of latency, discussed mechanisms of those, and discussed how much speedup is best for market efficiency.

Therefore, in this study we additionally implemented a latency model to the model of Mizuta et al. (2013b), and investigated price formations and market efficiency for various latency; other settings were exactly same. We tried to clarify mechanisms of those and discussed how much speedup is best for market efficiency. We also analyzed empirical data of Tokyo Stock Exchange, and compared empirical results with simulation results.

2   Artificial Market Model

An artificial market, which is a kind of a multi agent simulation, can isolate the pure contribution of these changing system to the price formation and can treat the changing that have never been employed (LeBaron (2006); Chen et al. (2012); Cristelli (2014)). These are strong points of the artificial market simulation study.

However, outputs of the artificial market simulation study would not be accurate and credible forecasts in actual future. It is an important role for the artificial market simulation to show possible mechanisms affecting a price formation by many simulation runs, e.g. a parameters search, purely comparing before/after the changing, and so on. The possible mechanisms showed by these simulation runs will give us new knowledge and intelligence about effects of the changing to price formation in actual financial market. Other study methods, e.g. empirical studies, would not show such possible mechanisms. It is most important purpose of the artificial market simulation studies to gain such new knowledge and intelligence.

Indeed, artificial markets should replicate macro phenomena existing generally for any asset and for any time, however, price variation which is a kind of macro phenomena is not explicitly modeled in artificial markets. Only micro processes, agents (general investors) and price determination mechanisms (financial exchanges) are explicitly modeled in artificial markets. Macro phenomena are emerged as the outcomes interactions of micro processes. Therefore, the simulation outputs should replicate general macro phenomena at least to show that simulation models are probable in actual markets.

However, it is not primary purpose for the artificial market to replicate specific macro phenom-

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[*7] More detail, Tokyo Stock Exchange (2013).

[*8] For example, Gsell (2009); Wang et al. (2013); Kusada et al. (2014).

[*9] For example, Wah and Wellman (2013).

ena only for a specific asset or a specific period. Replication of unnecessary macro phenomena for the study purpose leads over fitted and too complex models. Such over fitted and too complex models would prevent our understanding and discovering mechanisms affecting a price formation because of related factors in model increasing.

The purpose of this study is not accurate and credible forecasts in actual future but new knowledge and intelligence about effects of changing a market system to price formation in actual financial market by investigation many possible scenarios and mechanisms. This study discusses about actual system existing in actual financial markets, however, it is not purpose for this study to replicate quantitatively accurate price formation induced by these changing systems, but it is purpose to understand what possibly occurs by effects and/or side effects of the changing. Therefore, a price determination mechanism of this study model replicated actually financial market system on necessary parts to implement investigating regulations and system in this study, and agents were modeled as simply as replicating general macro phenomena. Because the purpose of this study is not replicating quantitatively accurate price formation, we do not intentionally implement agents to cover all the investors who would exist in actual financial markets.

Artificial models which are too complex and have too many parameters were often criticized because it is very difficult to evaluate models (Chen et al. (2012)). The too complex model not only would prevent our understanding and discovering mechanisms affecting a price formation, but also could output arbitrary result by over fitting of too many parameters. Models are simpler, it is more difficult to make arbitrary result, and it is easier to be evaluated. Models having unnecessary many parameters might output unrealistic and impossible results for actual financial market by over fitting of too many parameters, and it is very difficult to evaluate such complex model.

Due to the above reasons, in this study we built an artificial market model as simple as possible to achieve the purpose of this study. Because the purpose of this study is not replicating quantitatively accurate price formation, we do not intentionally replicate specific macro phenomena only for a specific asset or a specific period, and do not intentionally implement agents to cover all the investors who would exist in actual financial markets.

In many previous artificial market studies, the models were verified to see whether they could explain the stylized facts such as a fat-tail, volatility-clustering, and so on (LeBaron (2006); Chen et al. (2012); Cristelli (2014)). A fat-tail means that the kurtosis of price returns is positive. Volatility-clustering means that the square returns have positive autocorrelation, and the autocorrelation slowly decays as its time separation becomes larger. Empirical studies on the fat-tail and volatility-clustering have shown that both stylized facts exist statistically in almost all financial markets (LeBaron (2006); Chen et al. (2012); Cristelli (2014)).

Conversely, as many empirical studies, e.g. Sewell (2006) mentioned, only the fat-tail and volatility-clustering are stably observed for any asset and in any period because financial markets are generally instable. Indeed, for the fat-tail, the kurtosis of price returns is stably and significantly positive and for the volatility-clustering, the square returns have stably and significantly positive

autocorrelation, however, the magnitude of these values are instable and very different according to asset and/or period.

For the fat-tail, the kurtosis of price returns was observed very broad magnitude about $1 \sim 100$, and for the volatility-clustering, the autocorrelation of the square returns was also observed very broad magnitude about $0.01 \sim 0.2$ (Sewell (2006)).

Due to the above reasons, an artificial market model which purpose is to understand mechanisms affecting a price formation by changing regulations such as this study should replicate that these stylized facts values are significantly positive and within reasonable range as we mentioned. Because these stylized facts values are instable in actual financial market, it is not essential for the models to replicate specific values of stylized facts.

In this study, we additionally implemented a latency model as we explain details following section 2.1 to the model of Mizuta et al. (2013b).

The model of Mizuta et al. (2013b), on which our model is based, succeed at replicating high frequency micro structures such as the trade rates, cancel rates, one tick volatility, and so on, which were not replicated by the model of Chiarella and Iori (2002) by modulating that an variance of order prices were narrower and changed to normal distributed.

The model of Mizuta et al. (2013b) were verified by replicating long-term statistical characteristics observed in real financial markets, fat-tail and volatility-clustering, and replicating very short term micro structure, trades rates, cancel rates and standard deviations of returns for one tick. Therefore, the model is verified to investigate the effect by changing financial market system affecting to micro structure of price formation.

2.1   Time passing and Latency

The model treats only one risk asset and non-risk asset (cash). The number of agents is n. First, an agent 1 orders to buy or sell the risk asset; after that, an agent 2 orders; at $t = 3, 4, , , n$, agents $3, 4, , , n$ order respectively. After the last agent n orders, going back to the first step, the agent 1 orders, and agents $2, 3, , , , n$ order respectively, and this cycle is repeated.

Time t increases by δt when an agent orders to the market[*10]. δt is a time interval of occurring one order. In this study, we assumed order is occurred randomly and the occurring obeys Poisson process. Therefore, δt is determined by random variables of exponential distributed in an average δo [*11].

As we mentioned in section 1, in actual financial markets it exists that a "latency"; length of time required to match orders and to transport data. After information of order, which an investor had

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[*10] Note that time t passes even if no deals are done.

[*11] Many previous empirical studies for various financial assets (for example, Takayasu et al. (2002)) showed that a time interval of occurring one order roughly obeys an exponential distribution, however, also showed that some separation from the distribution.
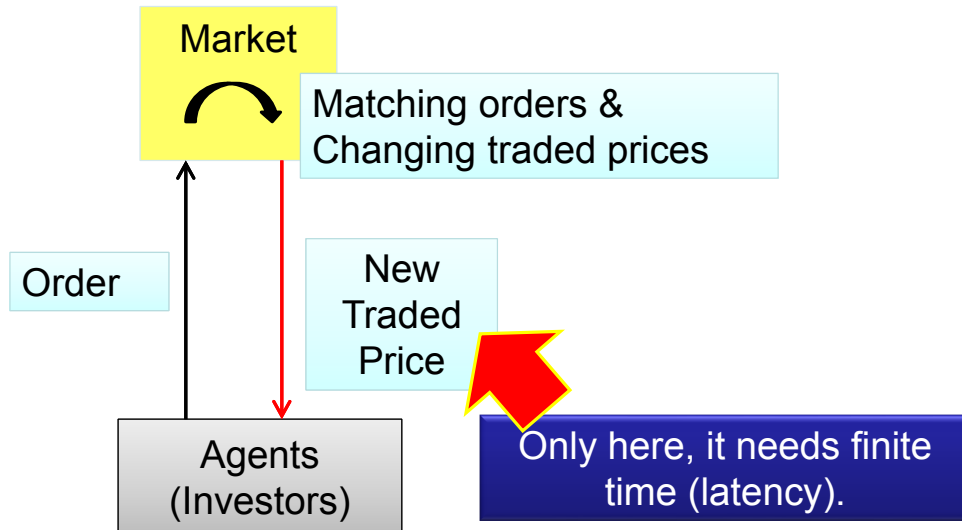
Figure 1　In this study, for simplify, finite latency only exist in transportation of traded price information from market system to an agent and the other latencies are zero.

made, has transported to market system, orders matched in market system. Furthermore, after matching orders and updating order book information, information of those has transported to the investor through a market information delivering system. Shortening latencies of a market system reduces time from an investor ordered to the investor received information of order book. In this study, for simplify, latencies are zero when agents order process, transportation of order information to market system, matching order process and updating information of order book. Finite latency only exist in transportation of traded price information from market system to an agent (Figure 1). In other word, agents observed a market price before time δl from true market price. Shorten δl reduces time difference between true price and observed price. We define P^t as agent's observed market price at time t.

It is a very important parameter that $\delta l/\delta o$; the ratio of a latency δl in an average order interval δo. Figure 2 shows diagrams relationship between time evolution of the observed prices and that of the true prices in the cases of $\delta l/\delta o > 1$ (top) and of $\delta l/\delta o \ll 1$ (bottom). In the case of $\delta l/\delta o > 1$ (top), it is sometimes occurred that $\delta l > \delta t$ (see the mid of yellow arrows). In this case, an agent might decide a different order from the case of the observed price being same as the true price. Therefore, in the case of $\delta l/\delta o > 1$, it is expected that price formation might be different from the case of no latency; $\delta l = 0$, because the observed is sometimes different from the true price, and a market system would be not enough fast. On the other hand, in the case of $\delta l/\delta o \ll 1$, it is almost $\delta l < \delta t$, and an agent usually knows the true price. Therefore, it is expected that price formation might be almost same from the case of no latency; $\delta l = 0$, and a market system would be enough fast.

In this study, we compared several statistical values of simulation runs for various $\delta l/\delta o$ under other parameters fixed.
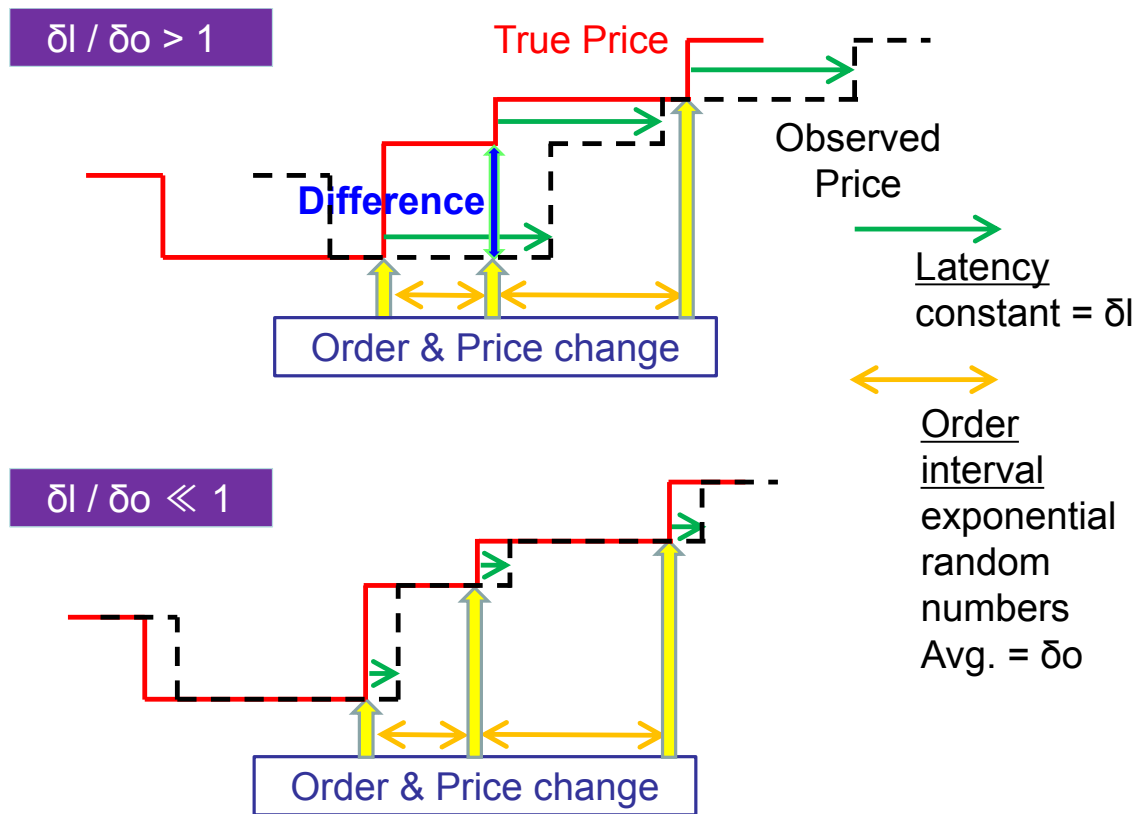
Figure 2　In the case of $\delta l/\delta o > 1$ (top), it is sometimes occurred that $\delta l > \delta t$ (see the mid of yellow arrows), and an agent might decide a different order from the case of the observed price being same as the true price. On the other hand, in the case of $\delta l/\delta o \ll 1$, it is almost $\delta l < \delta t$, and an agent usually knows the true price.

2.2　Trading Market

We adopted a continuous double auction to determine the market price of the risk asset. A continuous double auction is an auction mechanism where multiple buyers and sellers compete to buy and sell some financial assets in the market and where transactions can occur at any time whenever an offer to buy and an offer to sell match (Friedman (1993); Tokyo Stock Exchange (2012)).

A minimum unit of a price change (tick size) is δP, the buy order price is rounded off to the nearest fraction, and the sell order price is rounded up to the nearest fraction. Our model adopts the continuous double auction, so when an agent orders to buy (sell), if there is a lower price sell order (a higher price buy order) than the agent's order, dealing is immediately done, we call this order a "market order". If there is not, the agent's order remains in the order book, we call this order a "limit order"[*12].

[*12] Note that the definitions of a market order and of a limit order in this study are not exactly same as those in actual

2.3 Agents

When an agent j has a turn ordering, it determines an order price $P_{o,j}^t$ and buys or sells by the following process. Agents always order only one share. The quantity of holding positions is not limited, so agents can take any shares for long and short positions to infinity.

An agent j determines an order price and buys or sells by the following process. Agents use a combination of fundamental value and technical rules to form expectations on a risk asset's returns. An expected return of the agent j is

$$r_{e,j}^t = \frac{1}{w_{1,j} + w_{2,j} + u_j} \left(w_{1,j} \log \frac{P_f}{P^t} + w_{2,j} r_{h,j}^t + u_j \epsilon_j^t \right). \tag{1}$$

where $w_{i,j}$ is the weight of term i of the agent j, and is determined by random variables of uniformly distributed in the interval $(0, w_{i,max})$ at the start of the simulation independently for each agent. u_j is weight of third term of the agent j and is also determined by random variables uniformly distributed in the interval $(0, u_{max})$ at the start of the simulation independently for each agent. P_f is a fundamental value that is constant. P^t is a observed market price of the risk asset at time t as we mentioned section 2.1[*13]. ϵ_j^t is a noise determined by random variables of normal distribution with an average 0 and a variance σ_ϵ. $r_{h,j}^t$ is a historical price return inside an agent's time interval τ_j, and $r_{h,j}^t = \log(P^t/P^{t-\tau_j})$. τ_j is determined by random variables uniformly distributed in the interval $(1, \tau_{max})$ at the start of the simulation independently for each agent.

The first term of Eq. (1) represents a fundamental strategy: an agent expects a positive return when the market price is lower than the fundamental value, and vice versa. The second term of Eq. (1) represents a technical strategy: an agent expects a positive return when historical market return is positive, and vice versa.

After the expected return has been determined, an expected price is

$$P_{e,j}^t = P^t \exp(r_{e,j}^t). \tag{2}$$

An order price $P_{o,j}^t$ is determined by random variables of normal distributed in an average $P_{e,j}^t$, a standard deviation P_σ, where P_σ is a constant.

Buy or sell is determined by a magnitude relationship between the expect price $P_{e,j}^t$ and the order price $P_{o,j}^t$, that is,

$$\text{When } P_{e,j}^t > P_{o,j}^t, \text{ the agent orders to buy one share,}$$
$$\text{When } P_{e,j}^t < P_{o,j}^t, \text{ the agent orders to sell one share.} \tag{3}$$

The remaining limit order which an agent ordered c times before is canceled.

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financial markets.

[*13] When the dealing is not done at t, P^t remains at the last market price P^{t-1}, and at $t = 1$, $P^t = P_f$.

Figure 3 Volatilities and kurtoses of returns where returns within a time length, $\delta r = 1 \times \delta o$ for various latencies $\delta l/\delta o$.
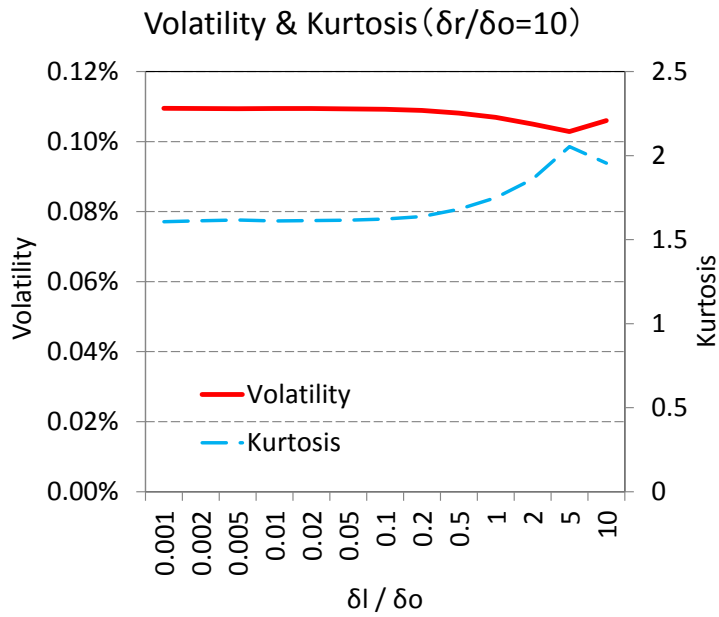
Figure 4 Volatilities and kurtoses of returns where returns within a time length, $\delta r = 10 \times \delta o$ for various latencies $\delta l/\delta o$.

3 Simulation Results

In this study, we set $\delta o = 1$ and the other parameters were same as those of Mizuta et al. (2013b)[*14]. Specifically, we set, $n = 1,000, w_{1,max} = 1, w_{2,max} = 10, u_{max} = 1, \tau_{max} = 10,000, \sigma_\epsilon =$

[*14] When $\delta l = 0$, our model and that of Mizuta et al. (2013b) are exactly same.
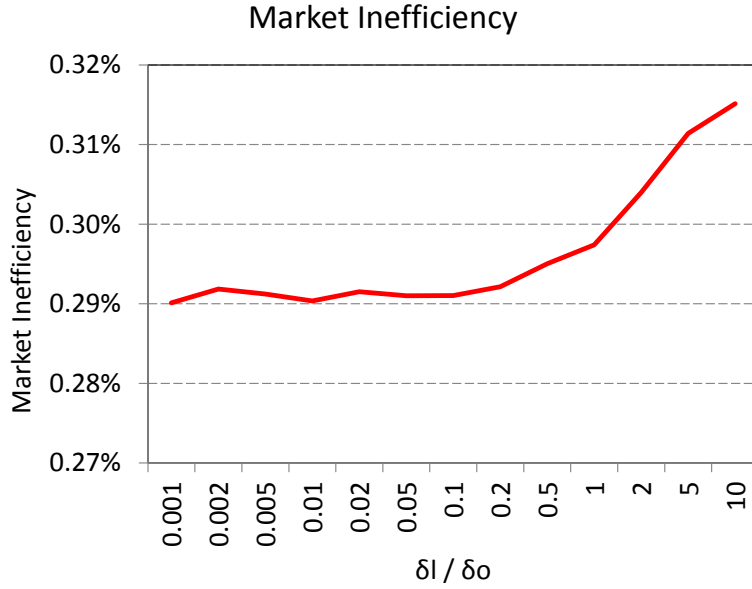
Figure 5　Market inefficiency M_{ie} for various $\delta l/\delta o$

$0.06, P_\sigma = 30, c = 20, \delta P = 0.1, P_f = 10,000$. We ran simulations until every agent orders $10,000$ times. We defined t_e as the end time of the simulation.

In this study, we compared several statistical values of simulation runs for various $\delta l/\delta o = 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10$ under not only other parameters are fixed but also we used same random number table. We simulated these runs 100 times changing the random number table and we used average statistical values of 100 times runs.

3.1　Volatility, Fat-tail and Market Inefficiency

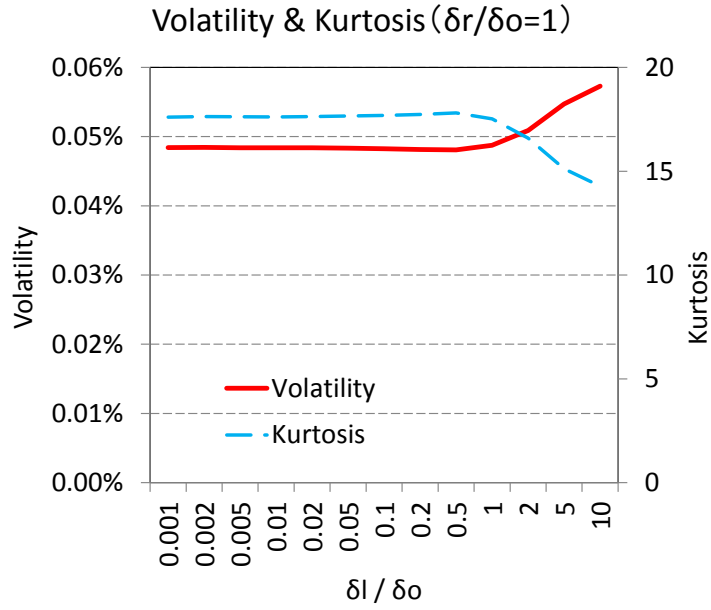Figure 3 shows volatilities (standard deviations of returns) and kurtoses of returns[*15] where returns within a time length, $\delta r = 1 \times \delta o$ for various latencies, $\delta l/\delta o$. In the left side, $\delta l/\delta o < 1$, both the volatilities and the kurtoses are stable, however, in the right side, $\delta l/\delta o > 1$, the volatilities are increasing and the kurtoses are declining.

Figure 4 shows as same as Figure 3 except returns within a time length, $\delta r = 10 \times \delta o$. In the right side, $\delta l/\delta o > 1$, the volatilities are slightly declining and the kurtoses are increasing. As seen above, we cannot judge whether market becomes efficient or inefficient from volatilities and kurtoses because they depend on returns within a time length.

Therefore, we introduced market inefficiency M_{ie} which directly measures market efficiency,

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[*15] Here, we used definition of the kurtosis K that $K = 1/(nS^4) \times \sum_{i=1}^{n}(x_i - X)^4 - 3$, where n is a number of data, $x_i(i = 1, , , n)$ are data, X is an average of data, and S is a standard deviation of data.
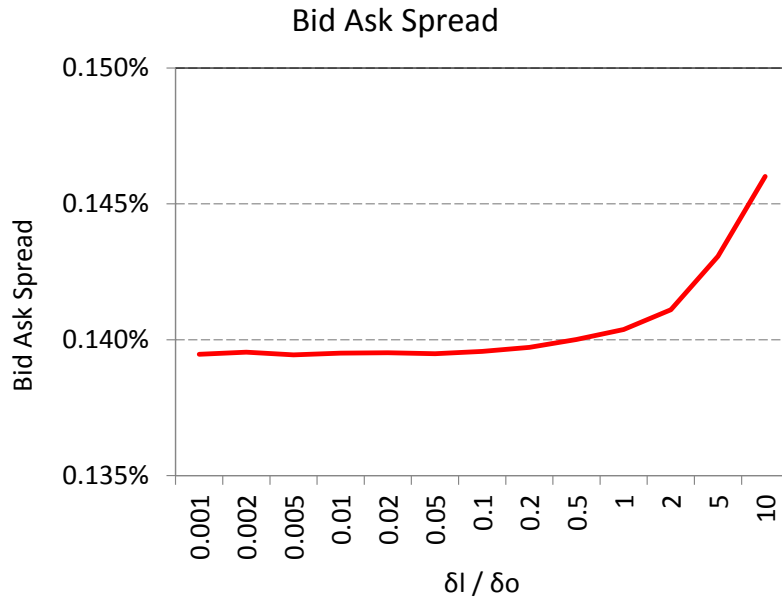
Figure 6　Bid ask spreads for various $\delta l/\delta o$.

and is sometimes used in experimental financial study for human beings[*16],

$$M_{ie} = \frac{1}{t_e} \sum_{t=1}^{t_e} \frac{|P^t - P_f|}{P_f}, \tag{4}$$

where ‖ means absolute value. M_{ie} is always greater than zero, and $M_{ie} = 0$ means a market is perfect efficient. M_{ie} is larger, a market is less efficient.

Many indications for measuring market efficiency have been proposed[*17]. Feature of M_{ie} is that M_{ie} is calculated by using a fundamental price P_f directly, which is never observed in empirical studies, and that we can use M_{ie} only in simulation and experimental studies. In simulation and experimental studies, we can calculate not estimated market efficiency but exactly market efficiency because we can exactly define a fundamental price.

Figure 5 shows market inefficiency M_{ie} for various $\delta l/\delta o$. In the left side $\delta l/\delta o < 0.5$, M_{ie} is very stable, however, in the right side $\delta l/\delta o > 0.5$, M_{ie} is larger which means the market is more inefficient. As we mentioned in section 2.1 and in Figure 2 when $\delta l/\delta o > 1$ the latency affects price formations, the effect makes a market inefficient. This indicates that it is needed $\delta l \ll \delta o$ that a market is efficient; a latency δl should be sufficiently smaller than an average order interval δo.

[*16] They sometimes call this market inefficiency "RAD" (Relative Absolute Deviation) in experimental financial studies. Stöckl et al. (2010) described details of RAD in experimental financial studies.
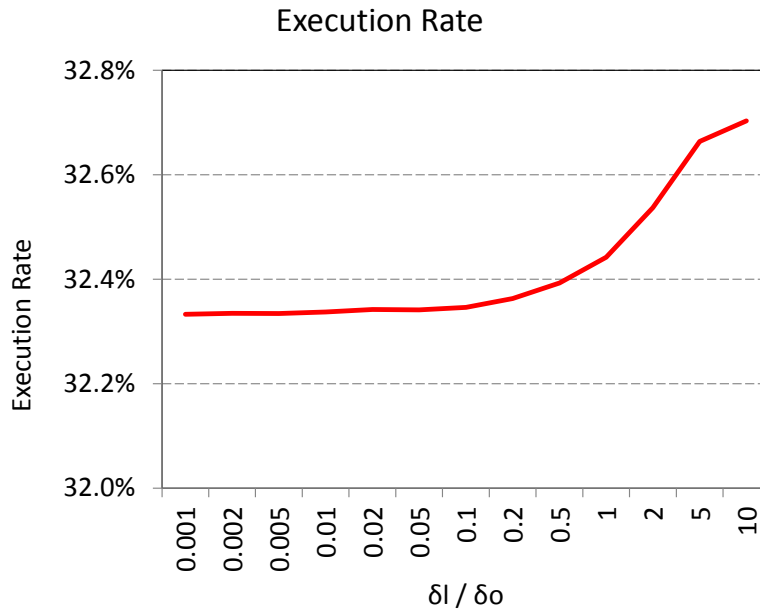
[*17] An excellent review; Verheyden et al. (2013).

Figure 7 Execution rates for various $\delta l/\delta o$.

3.2 Bid Ask Spread, Execution Rate

In this section, we discuss mechanisms that the latency is larger, a market is less efficient. Figure 6 shows bid ask spreads[*18] for various $\delta l/\delta o$. In the right side $\delta l/\delta o > 0.5$, bid ask spreads are wider.

Figure 7 shows execution rates for various $\delta l/\delta o$. We defined an execution rate is, number of market orders / number of all orders (market and limit orders). In the right side $\delta l/\delta o > 0.5$, execution rates are also increasing. These indicate that increasing execution rate reduces limit orders near the market price, and declining limit orders makes a bid ask spread wider, and wider bid ask spread makes a market less efficient. Next section, we discuss about a mechanism of the latency making execution rate larger.

3.3 Mechanism; Latency making Execution Rate Larger

In this section, we discuss the mechanism using results of each one run in the case of $\delta l/\delta o = 0.001$ and 10. Figure 8 shows execution rates for various true prices. In the case of large latency $\delta l/\delta o = 10$, the execution rates are larger than those in the case of no latency $\delta l/\delta o = 0.001$ especially near the fundamental price $P_f = 10,000$. Table 1 lists execution rates for the two cases, and averages of estimated return $r_{e,j}^t$ of all agents. We broke down the execution rates to the case of a market buy order matching a limit sell order and the case of a market sell order matching a

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[*18] We defined a bid ask spread S as $S = (P_{bb} - P_{ba})/P_f$, where P_{bb} is a highest limit buy order on an order book, and P_{ba} is a lowest limit sell order on an order book.
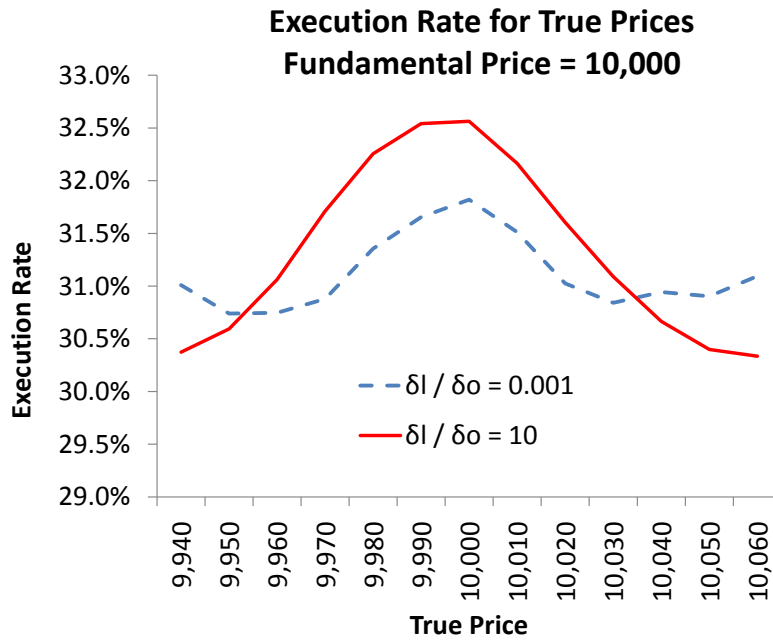
Figure 8 Execution rates for various true prices in the case of $\delta l/\delta o = 0.001$ and 10.

limit buy order. In the case of large latency $\delta l/\delta o = 10$, furthermore, we broke down them to the case of an observed price smaller than a true price and the case of an observed price larger than a true price. In the case of $\delta l/\delta o = 10$, when an observed price smaller than a true price there are more market buy orders than market sell orders and the average estimated return is positive, and vice versa.

Figure 9 shows diagrams describing the mechanism of the latency making execution rate larger to be consistent with results of Table 1. The equation (1) shows that, when a market price is near the fundamental price, $P^t \sim P_f$, the second term of equation (1) (the technical strategy term) is more dominant than the first term of equation (1) (the fundamental strategy term) in an estimated return of an agent $r_{e,j}^t$. The technical strategy term indicates positive (negative) estimated return when the historical return is positive (negative). In the left side of Figure 9 is the case of an observed price is smaller than a true price, the market price has upward momentum because that the observed price is older than the true price. Therefore, the technical strategy term is positive and the estimate returns of agents are frequently positive. Note that from (3) when the estimated price $P_{e,j}^t$ is very higher than the true price, a buy order tends to become market order because of an average of the order price $P_{o,j}^t$. Here, we discuss the case that the upward trend of the true price actually has been finished. If the agents knew the true price, their estimated returns would be almost zero because the technical strategy term would be almost zero, and they would not make market buy orders. However, the agents actually observe older prices and upward trend because of large latency, then, they actually make market buy orders. In the right side, the case of an observed price is larger than a true price, vice versa; the agents actually make market sell orders because of large latency even though downward trend has been finished based on the true

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Table 1　Execution rates for the two cases, and averages of estimated return $r^t_{e,j}$ of all agents. We broke down the execution rates to the case of a market buy order matching a limit sell order and the case of a market sell order matching a limit buy order. In the case of large latency $\delta l / \delta o = 10$, furthermore, we broke down them to the case of an observed price smaller than a true price and the case of an observed price larger than a true price.

$\delta l / \delta o$			Execution Rate			Avg. Estimated Return of agents
		Sum	Buy Market Sell Limit Orders	Sell Market Buy Limit Orders	
10	Observed P. < True P.	32.5%	28.9%	3.5%	0.28%
	Observed P. > True P.	32.5%	3.6%	28.9%	−0.27%
0.001	---	31.2%	15.6%	15.6%	0.00%

prices. These cases cause that the execution rate becomes larger as Table 1 and Figure 8 showed.

In this way, in the case of the latency is large, when market trend has been finished, the agents cannot quickly change their estimate prices, and then, the agents make unnecessary market following trades especially near the fundamental price, finally, the execution rate becomes larger. As we mentioned in section 3.2, increasing execution rate reduces limit orders near the market price, and declining limit orders makes a bid ask spread wider, and wider bid ask spread makes a market less efficient. This mechanism causes a market to be inefficient when the latency is large $\delta l/\delta o > 1$.

4　Empirical Study

In this section, we investigate the ratio of a latency δl in an average order interval δo from the empirical order data of Tokyo Stock Exchange, and compare the results with the simulation results.

4.1　Data

Table 2 lists $\delta l/\delta o$ for various period, before and after speedup by introducing arrowhead which started on January 2010, using order data of Tokyo Stock Exchange. We investigated 5 periods, we call period 1, period 2, period 3, period 4, period 5, and period 6. The period 1 was the result of Uno (2012), and from the period 2 to 6 are results of our study.

Analysis periods are as follow; the period 1 is within one month, December 2009 when was before arrowhead, the period 2 is from 2nd August 2010 to 18th November 2011 when was after

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（near Fundamental Price）

Observed Price < True Price

Observed Price > True Price

○ ⇐ Estimated P.

True P.

Observed P.

Upward trend

Too High Estimated P.
⇒ Market Buy order
    ↑ If agents knew True P.
       they did not order.

Too Low Estimated P.
⇒ Market Sell order
    ↑ If agents knew True P.
       they did not order.

Stop market trend

But, agents cannot change Estimate price, quickly

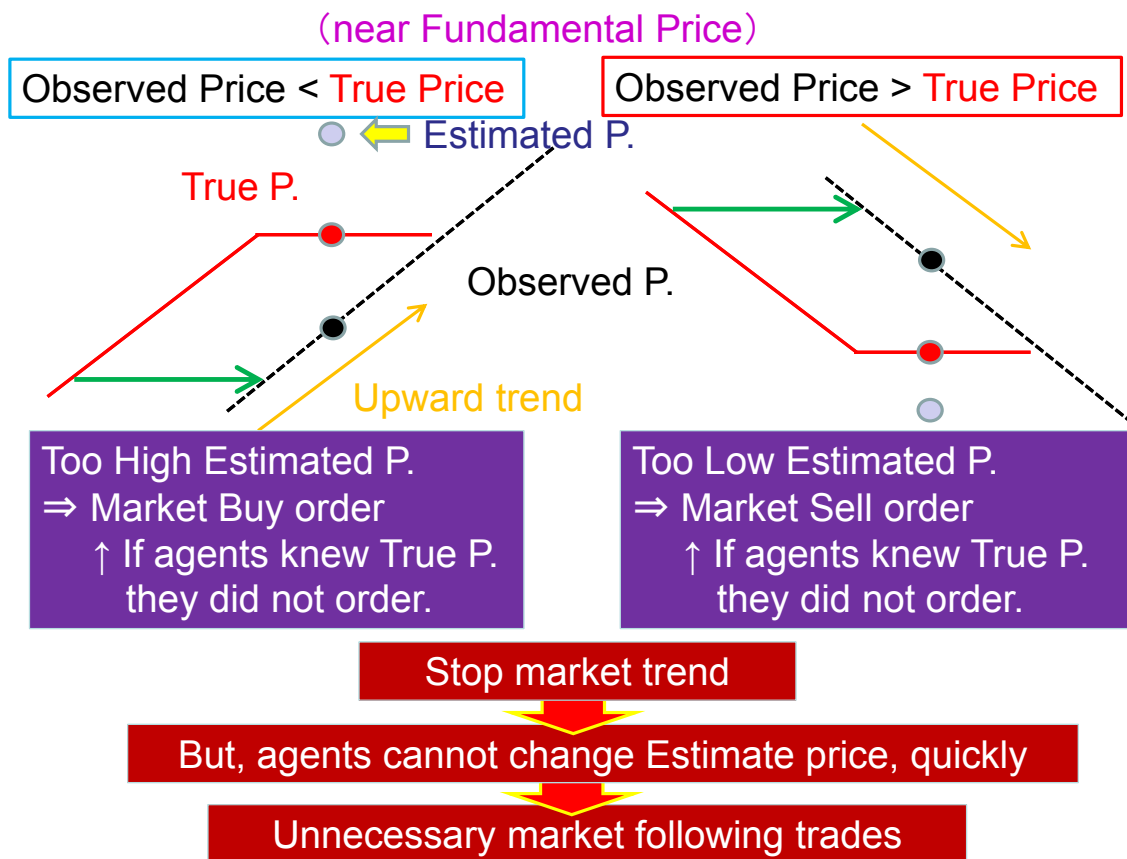Unnecessary market following trades

Figure 9   A diagrams describing the mechanism of the latency making execution rate larger. In the case of the latency is large, when market trend has been finished, the agents cannot quickly change their estimate prices, and then, the agents make unnecessary market following trades especially near the fundamental price, finally, the execution rate becomes larger.

arrowhead and before the extension of trading hour[*19], the period 3 is from 21th November 2011 to 26th November 2014 when was after arrowhead and after the extension of trading hour.

We also investigated the period when trading volume increased very much on latter 2014. On 31th October 2014 when Bank of Japan announced an expansion of the quantitative and qualitative monetary easing (Bank of Japan (2014)) trading volume increased very much, and on next business day, 4th November, the trading volume of first section recorded historical 2nd high at the time. The period 4 is from 27th October 2014 to 26th November 2014 including high trading volume period, the period 5 is on only one day, 31th October 2014 when Bank of Japan announced it, and the period 6 is only 4th November 2014 when the next business day of the announcement.

In the period 1, we analyzed stocks which were selected by TOPIX 100 at the time. In the period 2 to 6, we analyzed 81 stocks which were selected by TOPIX 100 on both start of 2009 and November 2014.

[*19] On 21th November 2011, trading hour of stocks in Tokyo Stock Exchange had been extended to 5 hours in all, 9:00-11:30 and 12:30-15:00 from 4 hours 30 minutes in all, 9:00-11:00 and 12:30-15:00.

Number of orders means number of new limit orders in the period 1[20], and number of all new orders in the period 2 to 6. In the each period, we used number averaged by both business days and analyzed stocks. In Tokyo Stock Exchange, there are opening auction sessions that investors can make orders but orders are not matched before stating trading hour[21]. In the period 1, calculation time span per one day is only trading sessions because the number of orders does not include orders within opening sessions, and in the period 2 to 6, calculation time span per one day is opening sessions and trading sessions because the number of orders includes orders within opening sessions. It is true that orders are not matched and market prices are not calculated in the opening sessions, however, investors' behaviors are same as in the trading sessions in terms that the market system receives orders, investors see order books, and they consider next orders.

In the period 1, we set latency δl 3 minutes which was a time span needed matching orders before introducing arrowhead[22], and in the period 2 to 6, we set latency δl 4.5 milliseconds which is summation of an average actual reply time, 2 milliseconds and an average actual delivery time, less than 2.5 millisecond showed by FTSE Global Markets (2010).

4.2   Result and Discussion

Table 2 shows $\delta l/\delta o$ is more than 0.5 in the period 1 before introducing arrowhead. As Figure 5 in the section 3 showed, simulation results indicated when $\delta l/\delta o$ is more than 0.5 the market is inefficient. Therefore, it is possible that the market was chronically inefficient, large portion of trading time, before arrowhead by the mechanism we mentioned in the section 3 (see also Figure 9).

On the other hand, in the period 2 to 6 after introducing arrowhead, Table shows $\delta l/\delta o \ll 1$ which indicates that the market is not chronically inefficient by the mechanism. The period 4, from 27th October 2014 to 26th November 2014, is well known as high trading volume period. Figure 10 shows $\delta l/\delta o$ for every business days in the period 4. $\delta l/\delta o$ on 31th October, 4th and 5th November were significantly larger than those of other days. Even though on these days, however, $\delta l/\delta o \ll 1$ were satisfied, and the market is not chronically inefficient by the mechanism.

Figure 11 shows $\delta l/\delta o$ every one minutes on 31th October, 4th and 5th November when $\delta l/\delta o$ were larger. Even $\delta l/\delta o$ every one minutes were less than 0.12 which was the max value, this indicates that the market is not chronically inefficient by the mechanism even for the time scale of minutes. Some minutes are enough short time spans to trade orders for almost investors except high frequency and/or high speed traders, and almost investors do not usually trade buy and sell

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[20] Of course, we should include market orders, however, here we show $\delta l/\delta o \ll 1$ even though not including market orders. (Note that δo is smaller than that of the case of including market orders.)

[21] Opening auction sessions exist for 85 minutes per one day, 60 minutes from 8:00 to 9:00, and 25 minutes from 12:05 to 12:30 (Tokyo Stock Exchange (2012)).

[22] A matching orders time span δo is not exactly same as the latency, however, they are same in term that investors cannot get latest information of an order book within the time.

Table 2 $\delta l/\delta o$ for various period, before and after speedup by introducing arrowhead which started on January 2010, using order data of Tokyo Stock Exchange.

No.	Analysis Period	arrowhead	Order No. Avg. for day Avg. names	Calculation Period (min)	Avg. δo (ms) = Period (ms) / Order No.	Latency δl (ms)	$\delta l / \delta o$
1	December 2009 (one month)	Before	2,833	270	5,718	3,000	0.525
2	2 August 2010 – 18 November 2011		14,621	355	1,457	4.5	0.003
3	21 November 2011 – 26 November 2014		28,974	385	797	4.5	0.006
4	27 October 2014 – 26 November 2014	After	66,044	385	350	4.5	0.013
5	31 October 2014 (one day)		87,109	385	265	4.5	0.017
6	4 November 2014 (one day)		114,027	385	203	4.5	0.022

many times in some minutes. Therefore, the market is not chronically inefficient for finite lengths of time for almost investors' trades by the mechanism simulation results showed in these days.

Figure 11 also shows, however, orders overcrowded at specific short time spans. Max number of orders recorded at 13:47 31th October after 3 minutes that Bank of Japan announced the expansion of the quantitative and qualitative monetary easing (Bank of Japan (2014)). Orders per one minute near this time were significantly more than other time. Like this, orders sometimes overcrowded for a very short term after very important announcements. In this study, we analyzed every one minutes, however, orders might overcrowded for shorter time spans. We cannot deny market inefficiency for less than one minute. For such shorter time spans, latencies δl might be instable, and the orders might overcrowded only to some specific stocks. These are future works. Such only short time order crowded would be very important for high frequency traders (HFTs) who orders buy and sell many times in very short time spans. For HFTs, it is very important that the market system is stably running and satisfying $\delta l/\delta o < 1$ when short time order crowded occurred after very important announcements.

Consequently, Tokyo Stock Exchange was possible that the market was chronically inefficient, large portion of trading time, before arrowhead by the mechanism simulation results showed. On the other hand, the market is not chronically inefficient by the mechanism at least for a time scale of minutes. It is a future work that the case of very crowded orders for less than one minute after announcements which are great market impacting information. The simulations of this study did not treat the case occurring very overcrowded orders for specific and very short spans, and simulation of the case is future work.

5 Conclusion and Discussion

In this study we additionally implemented a latency model to the model of Mizuta et al. (2013b), and investigated price formations and market efficiency for various latency; other settings were
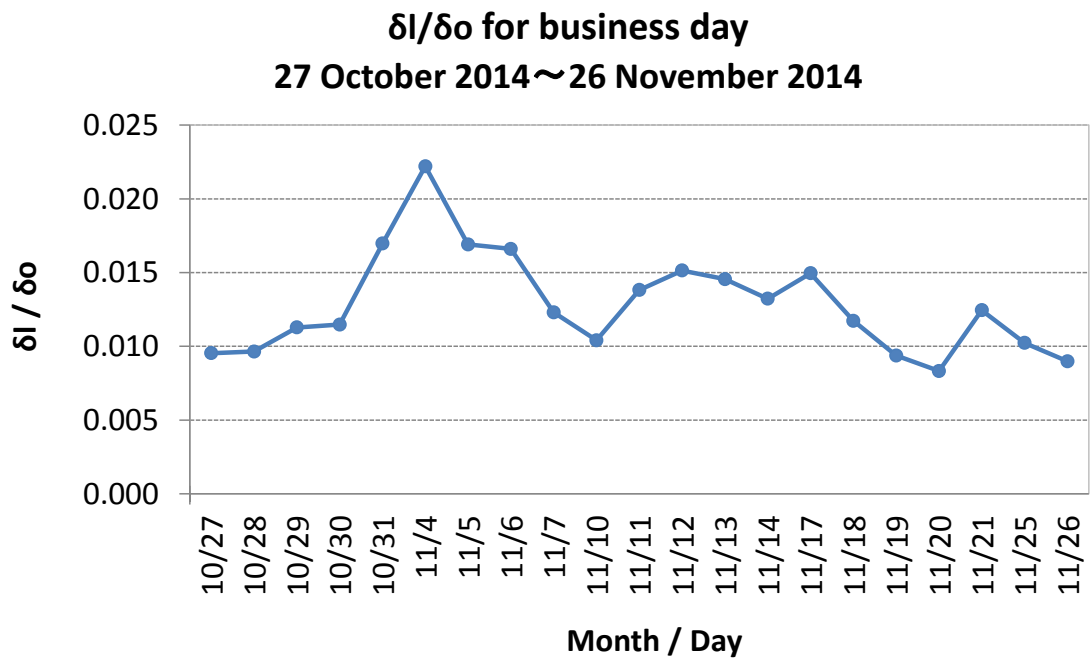
δl/δo for business day
27 October 2014～26 November 2014

Figure 10　$\delta l/\delta o$ from 27th October 2014 to 26th November 2014.

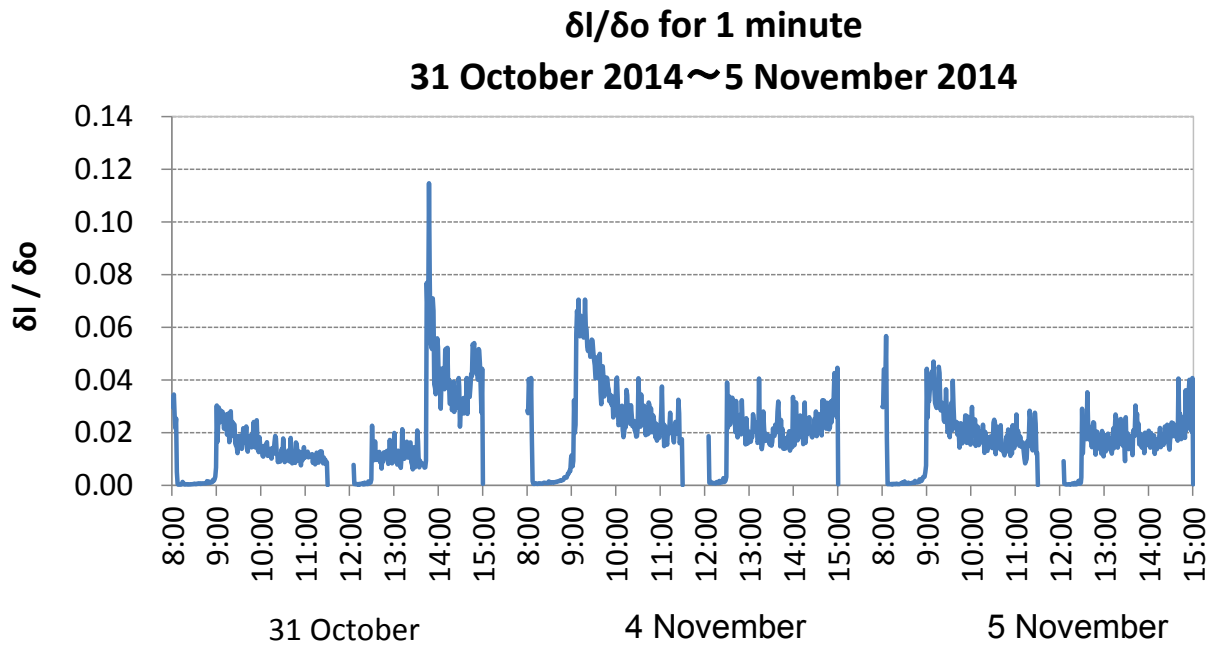**δl/δo for 1 minute**
31 October 2014～5 November 2014

Figure 11　$\delta l/\delta o$ every one minutes on 31th October, 4th and 5th November.

exactly same. We tried to clarify mechanisms of those and discussed how much speedup is best for market efficiency.

We showed that it is a very important parameter that $\delta l/\delta o$; the ratio of a latency δl in an average order interval δo. In the case of $\delta l/\delta o > 1$, we showed that increasing execution rate reduces limit orders near the market price, and declining limit orders makes a bid ask spread wider, and wider bid ask spread makes a market less efficient. This indicates that it is needed $\delta l \ll \delta o$ that a market is efficient; a latency δl should be sufficiently smaller than an average order interval δo. We also discussed about a mechanism of the latency making execution rate larger and showed that in the case of the latency is large, when market trend has been finished, the agents cannot quickly change their estimate prices, and then, the agents make unnecessary market following trades especially near the fundamental price, finally, the execution rate becomes larger.

Furthermore, we analyzed empirical order data of Tokyo Stock Exchange, and compare the results with the simulation results. Tokyo Stock Exchange was possible that the market was chronically inefficient, large portion of trading time, before arrowhead by the mechanism simulation results showed. On the other hand, the market is not chronically inefficient by the mechanism at least for a time scale of minutes.

It is a future work that the case of very crowded orders for less than one minute after announcements which are great market impacting information. The simulations of this study did not treat the case occurring very overcrowded orders for specific and very short spans, and simulation of the case is future work.

In this study we implemented only normal agents replicating general investors, however, the latency is more important especially for high frequency traders (HFTs) whose investment strategies are such as a market maker strategy, an arbitrage strategy, and so on. We should discuss about the latencies in more kinds of agents such them for future works. We did not directly investigate effects of HFTs, and this is also a future work.

As we mentioned, an artificial market can isolate the pure contribution of these new regulations to the price formation and can treat regulations and systems that have never been employed. However, outputs of the artificial market simulation study would not be accurate and credible forecasts in actual future. It is an important role for the artificial market simulation to show possible mechanisms affecting a price formation by many simulation runs and to gain new knowledge and intelligence; conversely, it is limitation of artificial market simulations that their outputs would not certainly but possibly occur in actual financial markets. Therefore, for more detail discussions, we should compare the simulation results to results of studies using other methods, e.g. empirical studies.

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